

Definite Integration

Properties of definite integration

Th 4. If  $f(x)$  is an even function then

$$\int_{-a}^a f(x) dx = 2 \int_a^a f(x) dx$$

A function that remains unchanged when  $x$  is replaced by  $-x$  i.e.  $f(-x) = f(x)$ , is called an even function. For example.

$\cos x, \sqrt{x^2 + a^2}, x \sin x$  etc.

Now  $\int_{-a}^a f(x) dx = \int_{-a}^0 f(x) dx + \int_0^a f(x) dx$

$x = -t$  in  $\int_{-a}^0 f(x) dx$

$dx = -dt$

when  $x = -a, t = a, x = 0, t = 0$

$\therefore - \int_a^0 f(-t) dt = I, \text{ (say)}$

$$\text{or, } I_1 = \int_0^a f(-x) dx$$

$$= \int_0^a f(-x) dx$$

$$= \int_0^a f(x) dx \text{ as } f(x) \text{ is an even function.}$$

$$\therefore \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

The 5 If  $f(x)$  is an odd function, then

$$\int_{-a}^a f(x) dx = 0$$

A function that changes sign when  $x$  is replaced by  $-x$  i.e.  $f(-x) = -f(x)$  is called an odd function. For example.  $\sin x$ ,  $x \cos x$  etc.

$$\begin{aligned} \text{Now } \int_{-a}^a f(x) dx &= \int_{-a}^a f(-x) dx \\ &= - \int_{-a}^a f(x) dx \text{ as} \end{aligned}$$

$f(x)$  is an odd function.

$$\therefore \int_{-a}^a f(x) dx = \int_{-a}^a f(x) dx + \int_0^a f(x) dx$$

$$= - \int_0^a f(x) dx + \int_0^a f(x) dx = 0$$

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Th. 6 If the lower limit of a definite integral is zero, then in the integrand the variable can be replaced by the upper limit minus the variable.

That is

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx.$$

Put  $x = a - y$ , then  $dx = -dy$ , when

$x = 0$ ,  $y = a$  and when  $x = a$ ,  $y = 0$

$$\therefore \int_0^a f(x) dx = - \int_a^0 f(a-y) dy$$

$$= \int_0^a f(a-y) dy.$$

$$= \int_0^a f(a-x) dx.$$